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Field redefinitions and wave function renormalization to $O(p^4)$ in heavy baryon chiral perturbation theory

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Abstract: Mass- and wave-function renormalization is calculated to order p^4 in heavy baryon chiral perturbation theory. Two different schemes used in the literature are considered. Several technical issues like field redefinitions, non-transformation of sources as well as subtleties related to the definition of the baryon propagator are discussed. The nucleon axial-vector coupling constant g_A is calculated to order p^4 as an illustrative example.

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Contents

1. Introduction

Heavy baryon chiral perturbation theory (HBChPT) [1, 2] allows for a systematic low energy expansion of one-nucleon Green functions. However, the matrix elements calculated in HBChPT are frame dependent. In order to obtain Lorentz invariant S-matrix elements, the fully relativistic nucleon propagator has to be worked out too. [3] So-called heavy nucleon sources cannot be neglected but yield non-trivial contributions to the nucleon wave function renormalization Z_{N} already at order p^3.

The frontier of HBChPT calculations presently lies at the order p^4 [4], and in one exceptional case at order p^5 [5]. Further complete p^4 calculations are needed in order to fully assess the convergence properties of the perturbative series. The aim of the present paper is to provide the renormalized parameters of the leading order chiral lagrangean, i.e. m_{N}, Z_{N} and g_A, to $\mathcal{O}(p^4)$ — a prerequisite for any such complete order p^4 calculations.

Two HBChPT lagrangeans widely used in the literature are considered. These are the lagrangean given in [2, 6] (called BKKM hereafter) and the form appearing in [7] (called EM). The difference consists in the absence of equation of motion (EOM) terms in EM, which have been eliminated by nucleon field redefinitions. The EM-form has the advantage of containing less terms. At order p^4 the reduction in complexity is already rather striking. Moreover, since EM uses a minimal basis of counter terms, the number of independent coupling constants can be inferred directly. On the other hand, the determination of the wave function renormalization constant Z_{N} is more involved, in particular when going beyond order p^3. We thus extend the work of ref. [3] and calculate mass and wave function renormalization to order p^4, both for the BKKM as well as the EM lagrangean. In the calculation of $Z_{\rm N}$ several additional issues enter compared to the treatment in [3].[1] The related subtleties are exposed by introducing a new EOM-transformation at the level of the relativistic lagrangean which allows for a direct and elegant evaluation of $Z_{\rm N}$. We also comment on the non-transformation of nucleon sources. Finally, we calculate the nucleon axial-vector coupling constant g_A to order p^4 in the two schemes considered. Although different at intermediate steps, the final results agree with each other, as expected. Some phenomenological implications of this result are discussed.

2. General formalism and EOM field transformations

The starting point for the derivation of HBChPT is the generating functional of relativistic Green functions

$$e^{iZ[j,\eta,\bar\eta]} = \mathcal{N}\int\left[dud\Psi d\bar\Psi\right]\exp\left\{i\left(\tilde S_{\rm M}+S_{\rm MB}+\int d^4x\left(\bar\eta\Psi+\bar\Psi\eta\right)\right)\right\}\,. \tag{2.1}$$

$j,\eta,\bar\eta$ denote the sources of mesonic and baryonic fields, respectively. $\tilde S_{\rm M}$ is the mesonic action — the tilde reminds us of the nucleon degrees of freedom having not been integrated out — and $S_{\rm MB}$ is the action corresponding to the pion nucleon lagrangean [8]

$$\mathcal{L}_{\pi N} = \bar\Psi\left(i\,\not{\nabla} - m + \frac{\dot g_A}{2}\,\not{u}\gamma_5\right)\Psi + \cdots\,, \tag{2.2}$$

where m and $\dot g_A$ denote the nucleon mass and axial-vector decay constant in the chiral limit, respectively. The ellipsis in (2.2) stand for higher order terms.

A systematic low energy expansion is obtained by the frame dependent decomposition of the nucleon field

$$\Psi(x) = e^{-imv\cdot x}(N_v+H_v)(x)\,, \tag{2.3}$$

with v being a unit time-like four-vector and

$$P_v^+N_v = N_v\,,\qquad P_v^-H_v = H_v\,,\qquad P_v^\pm = \frac{1}{2}(1\pm\not{v})\,. \tag{2.4}$$

In terms of these fields the pion-nucleon effective action may be rewritten as

$$S_{\rm MB} = \int d^4x\left\{\bar N_vAN_v + \bar H_vBN_v + \bar N_v\gamma_0B^\dagger\gamma_0H_v - \bar H_vCH_v\right\}\,. \tag{2.5}$$

Introducing sources corresponding to N_v, H_v

$$\rho = e^{imv\cdot x}P_v^+\eta\,,\qquad R = e^{imv\cdot x}P_v^-\eta\,, \tag{2.6}$$

[1] Wave function renormalization to order p^4 in the BKKM case was treated recently in [9]. The emphasis in this article is on different aspects than in the present work.

the heavy components H_v can be integrated out, [10, 2, 11] yielding

$$e^{iZ[j,\eta,\bar\eta]} = \mathcal{N}' \int \Big[dud N_v d\bar N_v\Big] \exp\Bigg\{ i\Big(\tilde S_{\rm M} + \int d^4x \Big[\bar N_v \tilde A N_v + \bar N_v \gamma_0 B^\dagger \gamma_0 C^{-1} R + \\ + \bar R C^{-1} B N_v + \bar N_v \rho + \bar\rho N_v + \bar R C^{-1} R\Big]\Big)\Bigg\}, \quad (2.7)$$

with

$$\tilde A = A + \gamma_0 B^\dagger \gamma_0 C^{-1} B\,. \quad (2.8)$$

Finally, expanding C^{-1} in a power series in $1/m$ and integrating over N_v yields the functional

$$e^{iZ[j,\eta,\bar\eta]} = \mathcal{N}'' \int [du] e^{i\,(S_{\rm M} + Z_{\rm MB}[u,j,\eta,\bar\eta])}\,, \quad (2.9)$$

where

$$Z_{\rm MB}[u,j,\eta,\bar\eta] = -\int d^4x \Big[(\bar\rho + \bar R C^{-1} B)\tilde A^{-1}(\rho + \gamma_0 B^\dagger \gamma_0 C^{-1} R) - \bar R C^{-1} R\Big]\,. \quad (2.10)$$

Any relativistic Green function is obtained from the functional (2.9) by taking derivatives with respect to appropriate sources. The simplest example of this kind is the two-point function of nucleon fields, which leads to a proper definition of the nucleon mass and wave function renormalization. For more details we refer the reader to ref. [3].

The matrices A,B,C occurring in (2.10) correspond to the effective action (2.5). Explicit expressions for these have appeared first in [2], and we therefore call this set of operators BKKM. An other form of the effective heavy baryon lagrangean was introduced in [7]. It was shown that so-called equation of motion terms in $\tilde A$ can be eliminated by a redefinition of the "light component field" N_v. However, in ref. [7] the effect of such field redefinitions was studied on the level of the effective lagrangean. The wave function renormalization, on the other hand, depends also on the operators B and C. What is the effect the EOM-transformations entail on B,C and hence on $Z_{\rm N}$?

In order to answer this question we re-investigate the nucleon field redefinitions on the level of the generating functional. We propose a variant of the EOM-transformations employed in [7], performed on the relativistic nucleon fields Ψ. Since the formalism of [3] emphasizes the relativistic nature of the problem, this seems to be more natural. We thus consider the field transformations

$$\Psi = (1 + e^{-imv\cdot x} T e^{imv\cdot x})\Psi'\,, \quad (2.11)$$

with

$$T = P_v^+ T_{++} P_v^+ + P_v^+ T_{+-} P_v^- + P_v^- T_{-+} P_v^+ + P_v^- T_{--} P_v^-\,. \quad (2.12)$$

The exponential factors in (2.11) are introduced such that possible derivatives in T act on N_v once the heavy baryon variables are introduced. The decomposition (2.12) is useful in order to separate the +/– sectors of the theory.

The same steps as performed in Eqs. (2.1)–(2.10) can be carried through provided we set

$$T_{+-} = T_{-+} = 0\,. \tag{2.13}$$

The generating functional then assumes the form (2.9), (2.10)[2] but with the replacements

$$\begin{aligned} A \to A' &= P_v^+(1+\gamma_0 T_{++}^\dagger \gamma_0)P_v^+ A P_v^+(1+T_{++})P_v^+ \\ B \to B' &= P_v^-(1+\gamma_0 T_{--}^\dagger \gamma_0)P_v^- B P_v^+(1+T_{++})P_v^+ \\ C \to C' &= P_v^-(1+\gamma_0 T_{--}^\dagger \gamma_0)P_v^- C P_v^-(1+T_{--})P_v^-\,, \end{aligned} \tag{2.14}$$

as well as

$$\begin{aligned} \rho \to \rho' &= P_v^+(1+\gamma_0 T_{++}^\dagger \gamma_0)P_v^+ \rho \\ R \to R' &= P_v^-(1+\gamma_0 T_{--}^\dagger \gamma_0)P_v^- R\,. \end{aligned} \tag{2.15}$$

Setting $T_{--} = 0$ and choosing T_{++} in accordance with the explicit expression given in [7], we recover the case where the EOM-transformations are performed on the fields N_v, *cf* [7]. The EOM-terms in the effective heavy baryon lagrangean $\tilde{A}' = \tilde{A}_{\rm EM}$ are then absent, by construction. However, according to (2.14) the matrix operator B is also changed. Explicit calculations reveal that the difference shows up first at $\mathcal{O}(p^4)$.

Consider now the EOM-transformations for general T_{--}. We still have $\tilde{A}' = \tilde{A}_{\rm EM}$, i.e. the effective lagrangean for the light component fields N_v' is the same. The T_{--} part of the transformation (2.11) therefore can be used to bring the factors $C'^{-1}B'$ in (2.10) to a convenient form. The point here is that the last term in (2.10), $\bar{R}'C'^{-1}R'$, does not exhibit poles and thus yields no contribution to S-matrix elements.

What is a convenient choice for T_{--}? In order to understand this question we have to add one further element to the discussion. We choose T_{--} such that the dressed nucleon propagator has standard form, i.e.

$$S_{\rm N}(p) = \frac{A(p^2)\not{p} + B(p^2)m_N}{p^2 - m_N^2}\,. \tag{2.16}$$

In general this need not to be true. In the present application, there is an other four-vector at our disposal, namely v_μ. The numerator of (2.16) may then contain also a term of the form $C(p^2)\not{v}$. This actually happens if we use the EOM-transformations (2.11) with (2.13) and $T_{--} = 0$ — the problem shows up first at order p^4. In this situation, one has to find the eigenvectors and corresponding eigenvalues of the dressed propagator in order to properly define the appropriate factors

[2] The Jacobian associated with change of variables (2.11) can be shown to yield no contribution to S-matrix elements.

for external legs. [12] This problem can be circumvented by exploiting the freedom to choose T_{--}, at least to order p^4. The point is that only the field independent part of the critical term $C^{-1}B$ is needed. Choosing

$$T_{--} = T_{++} \tag{2.17}$$

obviously yields

$$C'^{-1}B' = C^{-1}B + \text{field dependent terms}\,. \tag{2.18}$$

Explicit calculations then show that the dressed propagator to $\mathcal{O}(p^4)$ has standard form, hence the wave function renormalization can be calculated as in [3] (cf. sect. 4). We conclude that aside from the redefinition of sources, the EOM-transformation (2.11) together with (2.13), (2.17) leads to a generating functional (2.9) with effective πN lagrangean as constructed in [7] but is otherwise unchanged compared to the BKKM case.

Finally we would like to comment on the significance of transformed sources appearing in (2.15). The two-point function of nucleon fields, for instance, is obtained by taking functional derivatives of the generating functional with respect to the sources η,$\bar{\eta}$. However, after applying the EOM-transformation the generating functional is written in terms of transformed sources ρ', R' or, equivalently, in terms of η' with

$$\begin{aligned} \eta' &= \alpha\,\eta, \\ \alpha &= \left(1 + e^{-imv\cdot x}\left[P_v^+\gamma_0 T_{++}^\dagger\gamma_0 P_v^+ + P_v^-\gamma_0 T_{--}^\dagger\gamma_0 P_v^-\right] e^{imv\cdot x}\right)\,. \end{aligned} \tag{2.19}$$

The wave function renormalization as well as any Green function with two nucleon and arbitrary number of mesonic legs is therefore multiplied with additional factors $\alpha^\dagger\alpha$. When calculating S-matrix elements, these factors will be cancelled, however, because the Green function has to be multiplied with two inverse nucleon propagators.[3] The conclusion is that S-matrix elements are independent of the choice of sources. For practical purposes it is more convenient to use the transformed sources η',$\bar{\eta}'$ for in this case the factor α in (2.19) and its functional average is not needed explicitly. We shall follow this prescription when calculating $Z_{\rm N}$ to $\mathcal{O}(p^4)$ below.

3. Effective lagrangean to $\mathbf{O(p^4)}$

Here we recollect all the terms of the effective πN-lagrangean needed for the complete one-loop renormalization of the nucleon mass, wave function and axial-vector coupling constant g_A. The relevant terms of the effective $\pi\pi$-lagrangean are well known [13].

[3] This intuitive argument can be put on a more rigorous footing by considering in detail the generating functional in the path integral formulation.

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The leading order relativistic πN-lagrangean was given in (2.2). Higher order terms, corresponding to ellipses in (2.2), are

$$
\begin{aligned}
\mathcal{L}^{(2)}_{\pi\mathrm{N}} &= \overline{\Psi}\left\{c_1\langle\chi_+\rangle - \frac{c_2}{4m^2}\left(\langle u_\mu u_\nu\rangle \nabla^\mu\nabla^\nu + \mathrm{h.c.}\right) + \frac{c_3}{2}\langle u\cdot u\rangle + \frac{c_4}{4} i\sigma^{\mu\nu}\left[u_\mu, u_\nu\right]\right\}\Psi \\
\mathcal{L}^{(3)}_{\pi\mathrm{N}} &= \frac{b}{(4\pi F)^2}\overline{\Psi}\frac{1}{2}\langle\chi_+\rangle \not{u}\gamma_5\Psi \\
\mathcal{L}^{(4)}_{\pi\mathrm{N}} &= \frac{d}{m(4\pi F)^2}\overline{\Psi}\langle\chi_+\rangle^2\Psi\,, \qquad (3.1)
\end{aligned}
$$

where we have displayed only those terms contributing to our calculations. The LECs c_i as well as b and d are finite. Infinite parts of the 3rd and 4th order LECs (needed for cancellation of loop infinities) are added explicitly in the heavy baryon effective lagrangean. b and d are then renormalized LECs with renormalization scale equal to the pion mass.

The HBChPT effective lagrangean in the BKKM version reads

$$
\begin{aligned}
\widehat{\mathcal{L}}^{(2)}_{\mathrm{BKKM}} &= \bar{N}_v\left\{\frac{1}{2m}(v\cdot\nabla)^2 - \frac{1}{2m}\nabla\cdot\nabla - \frac{i\dot{g}_A}{2m}\{S\cdot\nabla, v\cdot u\} + c_1\langle\chi_+\rangle + \right. \qquad (3.2) \\
&\quad \left. + \left(c_2 - \frac{\dot{g}_A^2}{8m}\right)(v\cdot u)^2 + c_3 u\cdot u + \left(c_4 + \frac{1}{4m}\right) i\varepsilon^{\mu\nu\rho\sigma}u_\mu u_\nu v_\rho S_\sigma\right\}N_v \\
\widehat{\mathcal{L}}^{(3)}_{\mathrm{BKKM}} &= \bar{N}_v\left\{\frac{i}{4m^2}v\cdot\nabla\left((\nabla\cdot\nabla - v\cdot\nabla)^2\right) + \frac{b}{(4\pi F)^2}\langle\chi_+\rangle S\cdot u\right\}\, N_v \qquad (3.3) \\
\widehat{\mathcal{L}}^{(4)}_{\mathrm{BKKM}} &= \bar{N}_v\left\{\frac{1}{8m^3}(v\cdot\nabla)^2\left(\nabla\cdot\nabla - (v\cdot\nabla)^2\right) + \right. \qquad (3.4) \\
&\quad \left. + \frac{c_1}{4m^3}\left(\overleftarrow{\nabla}_\mu\langle\chi_+\rangle\nabla^\mu - v\cdot\overleftarrow{\nabla}\langle\chi_+\rangle v\cdot\nabla\right) + \frac{d}{m(4\pi F)^2}\langle\chi_+\rangle^2\right\}N_v\,,
\end{aligned}
$$

where only the finite part of the lagrangean was displayed. For infinite terms see [11] and [14].

As already mentioned, this effective lagrangean can be simplified considerably by the EOM-transformations, leading to the EM version of the HBChPT effective lagrangean. In the second order the simplification is only modest, but in higher orders it is more impressive

$$
\begin{aligned}
\widehat{\mathcal{L}}^{(2)}_{\mathrm{EM}} &= \bar{N}_v\frac{1}{m}\left\{-\frac{1}{2}\nabla\cdot\nabla - \frac{i\dot{g}_A}{2}\{S\cdot\nabla, v\cdot u\} + a_1\langle u\cdot u\rangle + \right. \\
&\quad \left. + a_2\left\langle(v\cdot u)^2\right\rangle + a_3\langle\chi_+\rangle + a_5 i\varepsilon^{\mu\nu\rho\sigma}u_\mu u_\nu v_\rho S_\sigma\right\}N_v \qquad (3.5) \\
\widehat{\mathcal{L}}^{(3)}_{\mathrm{EM}} &= \frac{\widehat{b}}{(4\pi F)^2}\bar{N}_v\langle\chi_+\rangle S\cdot u\, N_v \\
\widehat{\mathcal{L}}^{(4)}_{\mathrm{EM}} &= \frac{\widehat{d}}{m(4\pi F)^2}\bar{N}_v\langle\chi_+\rangle^2 N_v\,. \qquad (3.6)
\end{aligned}
$$

The relations between the LECs a_i and c_i are: $a_1 = \frac{1}{2} m c_3 + \frac{1}{16} \dot{g}_A^2$, $a_2 = \frac{1}{2} m c_2 - \frac{1}{8} \dot{g}_A^2$, $a_3 = m c_1$ and $a_5 = m c_4 + \frac{1}{4}(1 - \dot{g}_A^2)$. The LECs $\widehat{b}$ and $\widehat{d}$ are divergent

$$\widehat{b} = b - \frac{(4\pi F)^2}{m^2} \dot{g}_A a_3 + \left(\frac{1}{2}\dot{g}_A + \dot{g}_A^3\right) \left[\ln \frac{M}{\mu} + (4\pi)^2 L(\mu)\right] \tag{3.7}$$

$$\widehat{d} = d - \frac{(4\pi F)^2}{2m^2} a_3^2 - \frac{3}{16}\left(4a_1 + a_2 - 4a_3 + \frac{3}{8}\dot{g}_A^2\right) \left[\ln \frac{M}{\mu} + (4\pi)^2 L(\mu)\right], \tag{3.8}$$

where M is the pion mass to leading order in the chiral expansion and

$$L(\mu) = \frac{\mu^{D-4}}{(4\pi)^2} \left\{ \frac{1}{D-4} - \frac{1}{2}[\ln 4\pi + 1 + \Gamma'(1)] \right\} . \tag{3.9}$$

4. Nucleon mass and wave-function renormalization

Nucleon mass and wave-function renormalization are determined by the nucleon self energy. In the EM framework the one-loop (fig. 1) and tree graph contributions are given by

$$\Sigma^{\rm EM} = \Sigma^{\rm EM}_{\rm tree} + \Sigma^{\rm EM}_{\rm loop} \, , \tag{4.1}$$

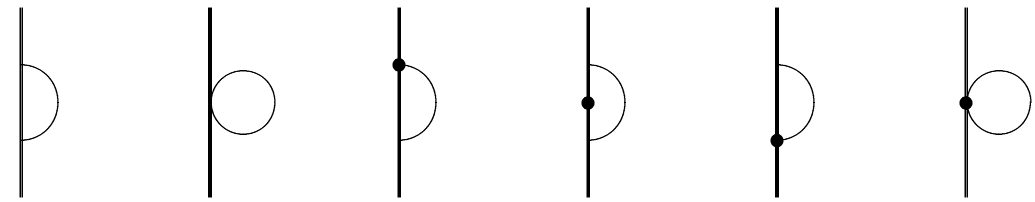
with

$$\Sigma^{\rm EM}_{\rm tree} = -\frac{k^2}{2m} - \frac{4M^2 a_3}{m} - \frac{M^4 \widehat{d}}{\pi^2 F^2 m} \tag{4.2}$$

$$\begin{aligned} \Sigma^{\rm EM}_{\rm loop} &= \frac{3\dot{g}_A^2}{4F^2}(D-1)\left\{ J_2(\omega) + \frac{M^2}{mD}\Delta + \frac{1}{2m}\left[k^2 + (1+8a_3)\, M^2 - 2\omega^2\right] J_2'(\omega) \right\} - \\ &\quad - \frac{6M^2}{mF^2}\left(a_1 + \frac{1}{D}a_2 - a_3\right)\Delta \, . \end{aligned} \tag{4.3}$$

k is the nucleon residual momentum defined by $p = m \cdot v + k$; $J_2(\omega)$, Δ are standard one-loop integrals explicitly given in e.g. [15] and $\omega = v \cdot k$.

To extract the nucleon mass and wave function renormalization, one has to find the position and the residue of the pole of the nucleon propagator. Surprisingly, this procedure is not as straightforward as one might expect and has become the subject



(a) (b) (c) (d) (e) (f)

Figure 1: One-loop diagrams contributing to the nucleon self-energy. Full circles are second order vertices.

of some discussions recently [3, 9]. The problem can be traced back to two simple facts: there are two propagators, the relativistic and the heavy baryon propagator. Moreover, the self-energy calculated in HBCHPT is a function of two scalar variables, ω and k^2. The relevant object to study is of course the relativistic propagator in the variable p^2 — HBChPT is, after all, just a particular way of organizing the perturbation series of the full relativistic theory. However, it requires some algebra to see that HBChPT calculations lead to the relativistic propagator.

First, we rewrite the full HBChPT propagator as a function of the variable p^2. To achieve this, we make use of the fact that the nucleon self-energy contains (both in BKKM and EM versions) the term $-\frac{1}{2m}(k^2+8M^2a_3)$. We can therefore write

$$\frac{i}{\omega-\Sigma}\equiv\frac{i}{\Omega-\Sigma_{\text{rest}}}\,, \tag{4.4}$$

where

$$\Omega=\omega+\frac{1}{2m}\left(k^2+8M^2a_3\right)=\frac{1}{2m}\left(p^2-m^2+8M^2a_3\right). \tag{4.5}$$

The next step is to trade ω for Ω also in Σ_{loop}. The crucial point here is that whenever a loop integral $J_n\left(\omega\right)$ appears in the result, the structure $\frac{1}{2m}\left(k^2+8M^2a_3\right)\frac{\partial}{\partial\omega}J_n\left(\omega\right)$ appears at higher order as well (explicit results (4.3) and (4.21) provide particular illustrations of this fact). The technical reason is that insertion of the second order counter term into the nucleon propagator inside the loop always contains $\frac{1}{2m}\left(k^2+8M^2a_3\right)$; the square of the propagator which enters due to this insertion can then be written as the derivative of the propagator with respect to ω. Using the expansion

$$\Sigma_{\text{loop}}\left(\Omega,k^2\right)=\Sigma_{\text{loop}}\left(\omega,k^2\right)+\frac{1}{2m}\left(k^2+8M^2a_3\right)\frac{\partial}{\partial\omega}\Sigma_{\text{loop}}\left(\omega,k^2\right)+\cdots \tag{4.6}$$

and the fact that the difference between ω and Ω is one order higher than the omegas themselves, (4.3) can be rewritten as

$$\begin{aligned}\Sigma^{\text{EM}}_{\text{loop}}=\;&\frac{3\dot{g}_A^2}{4F^2}(D-1)\left[J_2\left(\Omega\right)+\frac{1}{2m}\left(M^2-2\Omega^2\right)J_2'\left(\Omega\right)+\frac{1}{D}\frac{M^2}{m}\Delta\right]-\\&-\frac{6M^2}{mF^2}\left(a_1+\frac{1}{D}a_2-a_3\right)\Delta+\mathcal{O}\left(p^5\right)\,.\end{aligned} \tag{4.7}$$

Now one expands $\Sigma^{\text{EM}}_{\text{loop}}\left(\Omega\right)$ around the so-far unknown pole position Ω_p

$$\omega-\Sigma^{\text{EM}}=\Omega+\frac{M^4\widehat{d}}{\pi^2F^2m}-\Sigma^{\text{EM}}_{\text{loop}}\left(\Omega_p\right)-\Sigma'^{\text{EM}}_{\text{loop}}\left(\Omega_p\right)\left(\Omega-\Omega_p\right)+\cdots \tag{4.8}$$

This must vanish for $\Omega=\Omega_p$, i.e.

$$\Omega_p=\Sigma^{\text{EM}}_{\text{loop}}\left(\Omega_p\right)-\frac{M^4\widehat{d}}{\pi^2F^2m}\sim\mathcal{O}(p^3)\,. \tag{4.9}$$

Consequently, $\Sigma^{\rm EM}_{\rm loop}(\Omega_p) = \Sigma^{\rm EM}_{\rm loop}(0) + \mathcal{O}(p^5)$ and similarly for $\Sigma'^{\rm EM}_{\rm loop}(\Omega_p)$. One can therefore write

$$\frac{i}{\omega - \Sigma^{\rm EM}} = \frac{i2m\widehat{Z}}{p^2 - m_N^2} + \cdots \tag{4.10}$$

with

$$\begin{aligned} m_N^2 &= m^2 - 8M^2 a_3 - \frac{2M^4 \widehat{d}}{\pi^2 F^2} + 2m\Sigma^{\rm EM}_{\rm loop}(0) + \mathcal{O}(p^5)\,, \\ \widehat{Z} &= 1 + \Sigma'^{\rm EM}_{\rm loop}(0) + \mathcal{O}(p^4)\,. \end{aligned} \tag{4.11}$$

More explicitly we have

$$m_N = m - \frac{4M^2 a_3}{m} - \frac{M^3}{2(4\pi F)^2}\left[3\pi \dot{g}_A^2 + \left(32d - \frac{3}{2}a_2 + \frac{21\dot{g}_A^2}{16}\right)\frac{M}{m}\right] + \mathcal{O}(p^5) \tag{4.12}$$

$$\widehat{Z} = 1 - \frac{3\dot{g}_A^2 M^2}{2(4\pi F)^2}\left[1 + 3\left(\ln\frac{M}{\mu} + (4\pi)^2 L(\mu)\right) - \frac{3\pi}{2}\frac{M}{m}\right] + \mathcal{O}(p^4)\,. \tag{4.13}$$

Up to now we were dealing with the HBChPT nucleon propagator rather then the full relativistic one, i.e. in the notation of [3] we have considered only S_{++}. The full relativistic propagator is given by

$$S_N = P_v^+ S_{++} P_v^+ + P_v^+ S_{+-} P_v^- + P_v^- S_{-+} P_v^+ + P_v^- S_{--} P_v^-\,. \tag{4.14}$$

Proceeding along the lines of [3] one finds in the case of EOM transformations defined by (2.11)–(2.13) and (2.17)

$$\begin{aligned} P_v^+ S_{+-} P_v^- &= P_v^+ S_{++} \frac{\not{k}^\perp}{2m}\left(1 - \frac{\omega}{2m} + \frac{2M^2 a_3}{m^2}\right) P_v^- + \cdots \\ P_v^- S_{-+} P_v^+ &= P_v^- S_{++} \frac{\not{k}^\perp}{2m}\left(1 - \frac{\omega}{2m} + \frac{2M^2 a_3}{m^2}\right) P_v^+ + \cdots \\ P_v^- S_{--} P_v^- &= P_v^- S_{++} \frac{k^\perp \cdot k^\perp}{4m^2}\left(1 - \frac{\omega}{2m} + \frac{2M^2 a_3}{m^2}\right)^2 P_v^- + \cdots \end{aligned} \tag{4.15}$$

where ellipses stand for terms not contributing up to the fourth order and $\perp$ denotes perpendicular to v, i.e. $X^\perp = X - v(v \cdot X)$. Using the simple relation $X^\perp P_v^\pm = P_v^\mp X^\perp$ and the fact that ω is of the second chiral order, one obtains

$$S_N = S_{++}\left\{P_v^+ + \frac{\not{k}^\perp}{2m}\left(1 - \frac{\omega}{2m} + \frac{2M^2 a_3}{m^2}\right) + P_v^- \frac{k^\perp \cdot k^\perp}{4m^2} + \mathcal{O}(p^4)\right\}. \tag{4.16}$$

At this point we could continue as in [3] and write $p = p_N + \lambda r$, where p_N is the on-shell nucleon momentum and r is an arbitrary four-vector introduced to control the on-shell limit.[4] Here we employ another method, which appears to be

[4]We emphasize that in [3] a special choice $r = v$ was used in the calculations, but one can check explicitly that for (4.16) the on-shell limit is independent of r.

even simpler. We use yet another decomposition of the nucleon momentum, $p = m_N \cdot v + Q$. This implies $\not p + m_N = 2m_N P_v^+ + \not Q$. Moreover, on the mass shell one has $2m_N v\cdot Q + Q^2 = 0$, i.e. $v\cdot Q = \mathcal{O}(p^2)$, and therefore in the vicinity of the pole

$$\begin{aligned} S_N &= S_{++}\left\{\frac{\not p + m_N}{2m_N} - \frac{\not v\, v\cdot Q}{2m_N} + \frac{\not Q}{2m_N}\left(\frac{\delta m - v\cdot Q}{2m_N} + \frac{2M^2 a_3}{m_N^2}\right) + P_v^- \frac{Q^2}{4m_N^2} + \mathcal{O}(p^4)\right\} \\ &= S_{++}\left\{\frac{\not p + m_N}{2m_N} - \frac{\not v\, v\cdot Q}{2m_N} - \frac{\not Q\, v\cdot Q}{4m_N^2} + \frac{Q^2}{8m_N^2} - \frac{\not v\, Q^2}{8m_N^2} + \mathcal{O}(p^4)\right\}, \end{aligned} \tag{4.17}$$

where $\delta m = m_N - m$. Collecting terms we finally have

$$S_N = S_{++}\frac{\not p + m_N}{2m_N}\left(1 + \frac{Q^2}{4m_N^2}\right) + \mathcal{O}(p^4) = \frac{\not p + m_N}{p^2 - m_N^2}\widehat{Z}\left(1 - \frac{\delta m}{m_N}\right)\left(1 + \frac{Q^2}{4m_N^2}\right) + \cdots \tag{4.18}$$

The dots correspond to higher orders and/or to terms vanishing at the pole.

We have arrived at the full relativistic nucleon propagator in the form of the bare one, but with the bare mass replaced by the physical one, and with an overall multiplicative factor, which is nothing else but $Z_N^{\rm EM}$:

$$Z_N^{\rm EM} = \widehat{Z} - \frac{\delta m}{m_N} + \frac{Q^2}{4m_N^2} + \mathcal{O}(p^4)\,. \tag{4.19}$$

In the BKKM framework the tree graph contribution to the nucleon self-energy is

$$\Sigma^{\rm BKKM}_{\rm tree,fin} = -\frac{1}{2m}\left(k^2 - \omega^2\right)\left(1 - \frac{\omega}{2m} + \frac{\omega^2}{4m^2} + \frac{4M^2 c_1}{2m}\right) - 4M^2 c_1 - \frac{dM^4}{m\pi^2 F^2}\,, \tag{4.20}$$

where the subscript "fin" stands for the finite part of the tree contribution and we refrain from giving explicitly the lengthy expression for the infinite part. The loop graph contribution is, after cancellations of extra terms coming from the difference between $\widehat{\mathcal{L}}^{(2)}_{\rm BKKM}$ and $\widehat{\mathcal{L}}^{(2)}_{\rm EM}$,

$$\Sigma^{\rm BKKM}_{\rm loop} = \Sigma^{\rm EM}_{\rm loop}\,. \tag{4.21}$$

To proceed in analogy with the EM case one should rewrite $\Sigma^{\rm BKKM}$ as a function of Ω. However, in $\Sigma^{\rm BKKM}_{\rm tree,fin}$ it is impossible to get rid of ω completely. A simple trick to circumvent the problem is to replace $1 - \frac{\omega}{2m} + \frac{\omega^2}{4m^2}$ by $(1 + \frac{\omega}{2m})^{-1} + \mathcal{O}(p^3)$, yielding

$$\omega - \Sigma^{\rm BKKM}_{\rm tree,fin} = \left(1 + \frac{\omega}{2m}\right)^{-1}\left[\left(1 + \frac{4M^2 c_1}{2m}\right)\Omega - \frac{M^2 c_1}{m^2}\Omega^2 - \frac{16M^4 c_1^2}{2m} + \frac{dM^4}{m\pi^2 F^2}\right]. \tag{4.22}$$

From now on one can proceed as in the EM case. One obtains again (4.10) with m_N given by (4.12) and with $\widehat{Z}$ replaced by

$$\widehat{Z} \to \left[\widehat{Z} + \frac{9\dot{g}_A^2 M^2}{2(4\pi F)^2}\left(\ln\frac{M}{\mu} + (4\pi)^2 L(\mu)\right)\right]\left(1 + \frac{\omega}{2m}\right)\left(1 + \frac{\delta m}{2m}\right). \tag{4.23}$$

The terms in proportion to $\ln\frac{M}{\mu}$ and $L(\mu)$ in (4.23) come from infinite EOM terms in the BKKM lagrangean (not displayed explicitly in (3.3)). The two multiplicative factors on the RHS of (4.23) cancel two terms in (4.19), which finally leads to

$$Z_N^{\rm BKKM} = 1 - \frac{3\dot{g}_A^2 M^2}{2(4\pi F)^2} + \frac{9\pi\dot{g}_A^2 M^3}{4m(4\pi F)^2} + \mathcal{O}(p^4)\,. \tag{4.24}$$

This result agrees with the findings of [9].[5]

5. The nucleon axial-vector coupling constant g_A to $\mathcal{O}(p^4)$

So far we have calculated nucleon wave-function and mass renormalization up to the 4th chiral order. Renormalization of these parameters of the leading order chiral lagrangean is to be used in any complete one-loop HBChPT calculation. On the other hand, the leading order chiral lagrangean contains yet another parameter, namely the nucleon axial-vector coupling constant g_A, which will also enter any complete one-loop result. It is therefore equally worth to calculate the relation between bare and physical g_A up to the 4th order.

g_A receives contributions from both, tree and one-loop graphs. Moreover, at the order we are working, the wave function renormalization enters too. Working out g_A for the two lagrangeans considered thus provides a consistency check on our results for $Z_{\rm N}$.

In the heavy baryon formalism, the matrix element of the iso-vector axial-vector current is given by [3]

$$\begin{aligned}\left\langle p_{\rm out}\middle|\bar{q}\gamma^\mu\gamma_5\tau^a q\middle|p_{\rm in}\right\rangle = &\left(1-\frac{t}{4m_N^2}\right)^{-1}\bar{u}_+(p_{\rm out})\tau^a\,\times \\ &\times\left\{\left[2\left(1-\frac{t}{4m_N^2}\right)S^\mu - \frac{q\cdot S}{m_N}v^\mu\right]G_A(t) + \frac{q\cdot S}{2m_N^2}q^\mu G_P(t)\right\}u_+(p_{\rm in}).\end{aligned} \tag{5.1}$$

Concentrating on $g_A = G_A(0)$ we put $t=0$. Furthermore, we need only that part of the the form factor in proportion to S_μ. In particular, pion-pole diagrams are $\sim q^\mu$ and need not to be considered.

The relevant one-loop diagrams are those of fig. 1 with axial source hooked on in all possible places. Since the lagrangeans of EM and BKKM are different, individual diagrams will in general yield different results. We obtain for the sum of all one-loop graphs

$$\begin{aligned} g_A^{\rm loop,\,EM} = &\frac{M^2}{F^2}\left\{\frac{\dot{g}_A}{2}\left(\dot{g}_A^2-4\right)\left[L+\frac{1}{16\pi^2}\ln\frac{M}{\mu}\right]+\frac{\dot{g}_A^3}{32\pi^2}\right\}+ \\ &+\frac{M^3}{mF^2\pi}\left\{-\frac{\dot{g}_A^3}{192}+\frac{\dot{g}_A}{3}\left(-a_1+a_5+\frac{1}{8}\right)\right\}\,. \end{aligned} \tag{5.2}$$

[5]The result for $Z_N^{\rm BKKM}$ seems to be at variance with [3]. However, as explained in the Erratum [3], the third order BKKM lagrangean in [3] is not equivalent to our eq. (3.3).

In the BKKM case, there are additional loop-contributions due to the EOM-terms in $\widehat{\mathcal{L}}^{(2)}_{\rm BKKM}$, *cf* Eq. (3.2). Moreover, this lagrangean is written in terms of coupling constants c_i. This yields the difference

$$g_A^{\rm loop,\,BKKM} - g_A^{\rm loop,\,EM} = \frac{M^3 \dot{g}_A^3}{mF^2\pi}\,\frac{3}{32}\,. \tag{5.3}$$

The tree graph contribution is obtained from (3.3),(3.6), (3.7) and [11], yielding

$$g_A^{\rm tree,\,EM} = \dot{g}_A + \frac{\widehat{b}}{(4\pi F)^2}\,4M^2 \tag{5.4}$$

and

$$g_A^{\rm tree,\,BKKM} - g_A^{\rm tree,\,EM} = \frac{4M^2}{m^2}\,\dot{g}_A a_3 - \frac{9\dot{g}_A^3 M^2}{2(4\pi F)^2}\left(\ln\frac{M}{\mu} + (4\pi)^2 L(\mu)\right). \tag{5.5}$$

Applying wave function renormalization finally yields a third piece

$$g_A^{\rm Z_N,\,EM} = \dot{g}_A\,\left(Z_N^{\rm EM}(0) - 1\right), \tag{5.6}$$

with $Z_N^{\rm EM}$ given in (4.19). The difference between the two schemes here reads

$$g_A^{\rm Z_N,\,BKKM} - g_A^{\rm Z_N,\,EM} = -\frac{4M^2}{m^2}\,\dot{g}_A a_3 - \frac{M^3 \dot{g}_A^3}{mF^2\pi}\,\frac{3}{32} + \frac{9\dot{g}_A^3 M^2}{2(4\pi F)^2}\left(\ln\frac{M}{\mu} + (4\pi)^2 L(\mu)\right). \tag{5.7}$$

We observe that the differences in eqs. (5.3),(5.5) and (5.7) exactly cancel. Renormalizing the $\mathcal{O}(p^3)$ coupling constant $\widehat{b}$ according to (3.7) we find in both schemes

$$g_A = \dot{g}_A + \frac{M^2}{(4\pi F)^2}(4b - \dot{g}_A^3) + \frac{M^3}{m(4\pi F)^2}\,\frac{2\pi}{3}\left[\dot{g}_A(1 - 8a_1 + 8a_5) + \frac{11}{2}\dot{g}_A^3\right]. \tag{5.8}$$

Although different at intermediate steps, the final results for g_A agree with each other. This is the consistency check announced above.

The result (5.8) has important phenomenological consequences. While the constant b in the second term on the right hand side is presently unknown, a rather precise estimate for the term in proportion to M^3 can be given. The counter term coupling constants a_1,a_5 are constrained from the nucleon sigma term and πN-scattering threshold parameters [15, 16],

$$\begin{aligned} a_1 &= -2.6 \pm 0.7 \\ a_5 &= 3.3 \pm 0.8\,, \end{aligned} \tag{5.9}$$

where error bars significantly larger than in [15, 16] have been assigned. The reason for these larger error bars is twofold. First, the well known problem with unrealistic error bars of πN-scattering threshold parameters reflects itself in too optimistic error bars of a_1,a_5. Second, the values of a_1,a_5 were determined within the 3rd chiral order calculation and their higher order corrections are presently unknown. In order to account for both of these uncertainities, we take for the error bars a conservative estimate of 25 %. Expanding consistently to higher orders and employing input parameters $g_A = 1.26$, $M = 0.14$ GeV, $m = 0.939$ GeV and $F = 0.093$ GeV we obtain a positive large correction

$$\Delta g_A|_{M^3} = 0.32 \pm 0.05\,, \tag{5.10}$$

where the error bar is dominated by the assumed uncertainty in a_1,a_5. This surprisingly large correction implies that order p^3 calculations carry potentially large uncertainties due to the finite renormalization of $\dot{g}_A$ entering at next order in the chiral expansion.

6. Conclusions

We have calculated nucleon mass-, wave function and axial-vector coupling constant renormalization to $\mathcal{O}(p^4)$ in heavy baryon chiral perturbation theory. Relations (4.12), (4.19)/(4.24) and (5.8) between bare and correspondig physical values of these quantities are to be used in any complete one-loop HBChPT calculation.

Two lagrangeans widely used in the literature have been considered. These are the lagrangean given in [2, 6] and the form appearing in [7]. For the former case, we have confirmed the already known results for mass- and wave function renormalization. New results were obtained in the latter case, where the lagrangean is simpler because so-called EOM-terms have been eliminated by nucleon field redefinitions. In this case, however, wave function renormalization is more involved due to subtleties arising in conjunction with the field redefinitions. We have proposed a new EOM-field transformation, performed on the relativistic nucleon fields. This yields the effective heavy baryon lagrangean given in [7] but also allows for a simple and elegant evaluation of Z_{N}. Our result (4.19) for Z_{N} associated to the lagrangean given in [7] enables a systematic use of this simpler version of lagrangean in future calculations.

We have completed renormalization of the parameters of the leading order chiral lagrangean by calculating the nucleon axial-vector coupling constant g_A to order p^4 in the two schemes considered. In this way, we have also checked our previous results — although different at intermediate steps, the final result for g_A is the same. Phenomenologically, the order M_π^3 correction to g_A turns out to be rather large, $\Delta g_A|_{M_\pi^3} \simeq 0.32$. This might have important phenomenological consequences when going beyond order p^3 in HBChPT.

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